A PSEUDO-BOUNDARY METHOD AND ITS APPLICATION TO INTERACTING CIRCULAR INHOMOGENEITIES IN A MULTIPLY CONNECTED POLYGONAL DOMAIN

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A numerical procedure is presented for solving two-dimensional elastostatics problems with large numbers of circular holes and elastic inclusions in a finite domain with a polygonal boundary. This problem has direct application in determining the effective elastic properties of micro-porous materials and fiber-reinforced composites. The inclusions may have arbitrary elastic properties, different from those of the matrix, and the holes may be traction free or loaded with uniformly distributed normal pressure. The loading can be applied on all or part of the finite external boundary.

Instead of solving the problem in a polygonal domain directly, we construct a pseudo circular boundary containing the polygonal region of interest and then apply loading on this boundary to satisfy the prescribed conditions on the polygon. The subsequent analysis is based on a semi-analytical solution obtained by the authors [1] for multiple circular inclusions and holes in a multiply connected circular domain using a combination of the series expansion technique with a direct boundary integral method in which complex potentials are written directly in terms of the actual boundary parameters [2]. For cases in which all geometric features are circular, all of the unknown boundary functions are approximated by truncated Fourier series. The elastic fields in the solids can be expressed analytically through the coefficients in the series expansions. Finally, the boundary conditions on the actual polygonal boundary are satisfied by adopting an over-specification technique based on a least squares approximation.

An iterative algorithm is developed to implement the numerical scheme outlined above. Convergence of the algorithm in a small number of iterations is achieved by suitable placement of the pseudo circular boundary — neither too far nor too close to the actual boundary. Global approximation of the boundary parameters without boundary discretization and analytical evaluation of all the integrals yields low computational cost and high accuracy of the numerical results. Several numerical experiments are described to demonstrate the effectiveness of the method for problems involving a rectangular domain.

A simplified model for a rectangular body containing multiple circular holes and elastic inclusions is also proposed to estimate the effective properties of multiphase fiber-reinforced composite materials. The capability of modeling a finite rectangular boundary will make the method useful for designing laboratory experiments for direct measurement of effective elastic properties.

References

[1] J. Wang, S.G. Mogilevskaya, and S.L. Crouch, "A Numerical Procedure for Multiple Circular Inclusions and Holes in a Finite Domain with a Circular Boundary," *Computational Mechanics*, to appear, 2003.
[2] A.M. Linkov and S.G. Mogilevskaya, "Complex hypersingular integrals and integral equations in plane elasticity," *Acta Mechanica*, v. 105, p. 189–205, 1994.